

# Quantum group conjecture and Stripes

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## Abstract

Keeping in mind our conjecture of modelling cuprates and related systems by quantum groups and our claim that the existence of stripes is intimately related to quantum groups and more strongly is a consequence of the underlying quantum group structure we note that a recent claim that *magnetic fluctuations* also displays a one-dimensional character makes our conjecture more viable. The recent strong claim by Mook et al., if true further supports our intuition. For the fluctuations associated with a striped phase are expected to be one-dimensional, whereas the magnetic fluctuations are *taken* to display two-dimensional symmetry. It is claimed by Mook et al. that this apparent two-dimensionality results from measurements on twinned crystals, and that similar measurements on substantial detwinned crystals of  $\text{YBa}_2\text{Cu}_3\text{O}_{6.6}$  reveal the one-dimensional character of the magnetic fluctuations thus making the stripe scenario more stronger, in other words there are also magnetic stripes. As noted before in our conjecture that one of the main reason for quantum groups to model cuprates is that they are directly related to the symmetries of one-dimensional systems. Thus now if the magnetic fluctuations are also shown exhibiting one-dimensional behavior that all the more *directly strengthens our conjecture*. However we note that our conjecture is more generic and not tied to the one-dimensional character of the now claimed magnetic fluctuations. Simply the one-dimensional nature of magnetic fluctuations as noted by Mook et al., makes our conjecture more *obvious* and clearer.

In a previous work one of us [1] have advanced the conjecture that one should attempt to model the phenomena of antiferromagnetism and superconductivity by using quantum symmetry group. Following this conjecture to model the phenomena of antiferromagnetism and superconductivity by quantum symmetry groups, three toy models were proposed [2], namely, one based on  $SO_q(3)$  the other two constructed with the  $SO_q(4)$  and  $SO_q(5)$  quantum groups. Possible motivations and rationale for these choices were outlined. In [3] a model to describe quantum liquids in transition from 1d to 2d dimensional crossover using quantum groups was outlined. Recently we [4] also proposed the particular choice of classical group  $SO(7)$  to incorporate the phenomenon of pseudo-gap not addressed in the  $SO(5)$  model. In [5] we considered an idea to construct a theory based on patching critical points so as to simulate the behavior of systems such as cuprates and specific system was cited as an example.

In this short note we point out the recent experimental work by Mook et al. [6] which claims the one dimensional nature of magnetic fluctuations in the  $YBa_2Cu_3O_{6.6}$  system and quantum group conjecture. Unlike other works where striped phases have been suggested to account for the properties of high  $T_c$  copper oxide materials. In our formulation the existence of stripes is intimately connected with the quantum groups.

Stripes may be considered as inhomogeneous distributions of charge and spin. Naively if charge and spin are confined to separate linear regions we can say that the system is in a striped phase. The separation of spin and charge as originally proposed by Anderson is linked to the formation of the stripe phase. The separation of spin and charge in turn is intimately related to quantum group. The quantum group allows the system to be broken into 1-d quantum liquids.

From the neutron scattering data one can see the existence of both the spin and charge fluctuations. These fluctuations of spin and charge can be taken to imply the existence of a

dynamic striped phase for  $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$  materials with high  $T_c$  value. However it has been noted that for two of the cuprates namely  $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$  and  $\text{La}_{2-x}\text{Sr}_x\text{Cu}_3\text{O}_{7-x}$  the magnetic fluctuations arising from spin fluctuations seem to exhibit a four-fold pattern around the incommensurate points around the  $(1/2, 1/2)$  reciprocal lattice positions. However Mook et al. [6], have recently claimed that this two-dimensional symmetry exhibited by magnetic fluctuations is a result of using *twinned* crystals in the neutron scattering studies. For if the crystal is substantially detwinned the magnetic fluctuations becomes *one-dimensional*. This means that not only the charge but spin fluctuations exhibit a one-dimensional character which further directly strengthens the quantum group scenario. Moreover as pointed out earlier [1–3] to get the complete picture of cuprates one may have to go beyond even the quantum group picture. As we patch the 1-d sub-systems which are a consequence of the quantum group symmetry to obtain the 2-d and 3-d systems we arrive naturally at a string picture. The sequence of a possible scenario is quantum group symmetry yields the 1-d system, the quantum groups are also related to Kac-Moody algebras which in turn are the generators of special non-linear sigma models, which are themselves an approximate form of strings [7,4,5].

Considering this scenario we note that in approximation it suffices that one can considers the interactions of spinons and holons. However one may regard spinons and holons as modes of a string and so in a more general picture one must expand the string [stripe] in term of the basic components such as spinons and holons. It is then possible to regard that as the strings [stripes] interact from the 2-d point of view the quantum group symmetry is ‘broken’.

Most people have tended to regard it a problem that quantum groups were closely tied with 1-d systems. However we take the contrary point of view, namely it is the close relationship between the 1-d systems and quantum groups which is useful in understanding

the cuprates and related systems since the basic fluctuations in these systems are 1-d.

The interaction of charge fluctuations with phonons is highly non-trivial. The charge scattering as imaged by phonons was reported in [8,6] for  $\text{YBa}_2\text{Cu}_3\text{O}_{6.6}$  using twinned crystals in a region of momentum space where it is expected a strong interactions between phonons and the striped phase. It is seen that the broad line shape [due to charge fluctuations] can be considered to be made of two ‘lines’ one from **b** twin domain [which is a normal narrow line] and a broad line from **a** twin domain that contains the striped phase 1-d modulation vector. This is consistent with the experimental findings mentioned in [2] about charge stripes and the phonon anomaly.

In conclusion experiments seem to indicate the one-dimensional nature of both charge and magnetic fluctuations. The one-dimensional nature of fluctuations is a consequence of quantum group symmetry. Experimental evidence suggests that the superconductivity in the cuprates is strongly influenced by the one-dimensional stripe phases. Thus quantum group symmetry must play a role in the understanding HTSC phenomenon.

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